Optimized Performance of a Semipassive Aerodynamic Controller

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A semipassive, aerodynamic attitude controller is proposed for spacecrafts in arbitrary near-Earth orbits. The analysis suggests that a system of flaps, operating in the free molecular environment, can efficiently damp the librational motion and provide a mechanism for imparting to the satellite any arbitrary orientation in space. The control performance is optimized, numerically, on the basis of the damping time in circular orbits and the maximum pointing error due to steady state limit cycles in elliptic trajectories. A parametric analysis yields the operating region for quick damping ($<\frac{1}{3}$ orbit) and small pointing errors ($<1°$). The controller promises to be economical, efficient, and would extend the useful life of near-Earth space vehicles.

Nomenclature

e	= eccentricity of orbit
f_ψ, f_ϕ	= planar and transverse components of aerodynamic moment due to flap rotations
r_p, R	= perigee distance and Earth-radius, respectively
v	= orbital velocity
x, y, z	= principal body coordinates with z along the axis of symmetry
A	= position coefficient, Eq. (2)
B_f	= aerodynamic coefficient for satellite without flaps
C_1	= ratio of transverse to axial cross-sectional areas of satellite, $\pi D_0/4L_0$
C_D, C_L	= drag and lift coefficients, respectively
D_0, L_0	= cylindrical satellite's diameter and length, respectively
K_i	= inertia parameter, $(I-I_{zz})/I$, $I = I_{xx} = I_{yy} > I_{zz}$
O_i	= limiter output, Fig. 7
S_i, C_i, T_i	= sin (i), cos (i), tan (i), respectively
T	= time for damping the transient motion
α_i	= flap rotations with respect to satellite
γ_i	= position control angle
Δ	= maximum pointing error during steady state
ε	= distance between satellite's center of pressure and center of mass, both assumed on z-axis
θ	= orbital position of satellite, measured from perigee
λ	= librational angle about z-axis (yaw); also eigen values in Eq. (6)
μ, ν	= proportionality constants in controller characteristic relations
ρ	= atmospheric density
τ	= control torque
ψ	= planar librational angle (pitch)
ϕ	= transverse librational angle (roll)

Subscripts and superscripts

e	= value of parameter at stable equilibrium configuration
0	= initial condition
p	= value of parameter at perigee
*,max	= maximum value
ψ, ϕ	= planar and transverse components
$(\ )'$	= differentiation with respect to θ

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Introduction

ATTITUDE dynamics of satellites employing passive stabilization procedures has been a subject of considerable study.[1] In general, one of the constraints of passive attitude control schemes has been the availability of rather small corrective torques thus making a system overly susceptible to the disturbing influence of environmental forces.[2] For example, the stability region for a gravity-gradient satellite is reduced significantly by solar radiation pressure at high altitudes[3,4] and by aerodynamic forces at lower altitudes.[5,6] This has led to investigations aimed at minimizing the adverse effects of environmental forces with recent attempts at utilizing them to advantage. Modi et al.[7,8,4] have proposed several semipassive solar controllers which not only damp the librational motion but also enable a satellite to attain any arbitrary spatial orientation.

Satellites with missions requiring near-Earth trajectories are subjected to atmospheric forces. Utilization of these forces in attitude control was a subject of several early discussions.[9,10] In practice, however, atmospheric forces have been successfully used[11] only for the pitch control of COSMOS-149, with other degrees governed by gyroscopic forces. In a model proposed by Hoffer,[12] the gyros were replaced by a set of moving masses. However, the power requirement in the former limits its useful life while the latter involves large inertia variations to be effective. A recent study by Ravindran[13] optimizes, through linearization, a set of controller flaps for a satellite in a circular orbit. Modi and Shrivastava,[6] on the other hand, have proposed several schemes for semipassive, velocity sensitive, aerodynamic controllers. Their nonlinear analysis showed the aerodynamic controller to be effective in damping the effect of severe disturbances in a fraction of an orbit. The performance of the controller appeared promising even in elliptic orbits where the corrective moments may be available only over a portion of the trajectory.

This paper investigates librational damping and attitude control of an axisymmetric satellite in an arbitrary orbit using a velocity and displacement sensitive aerodynamic controller. In the beginning, equilibrium configurations of the system are studied as functions of system parameters and controller gains, and their stability in the small established. This is followed by an extensive optimization study based on transient (autonomous) and steady-state (nonautonomous) behavior of the system. Typical response plots illustrate the satellite's ability to attain any desired spatial orientation through an appropriate choice of the controller characteristics. A preliminary estimate of the power requirement and attainable pointing accuracy are included followed by a brief discussion of the feasibility of a solar-

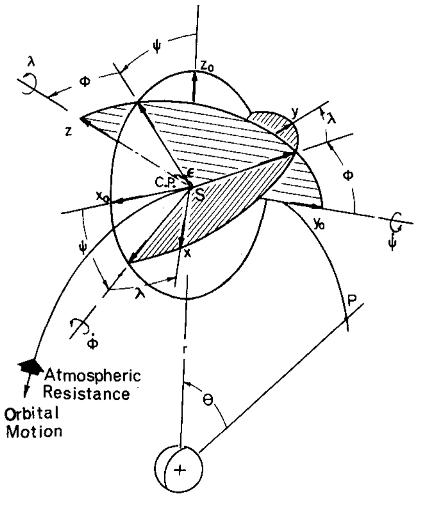

Fig. 1 Geometry of satellite motion.

aerodynamic hybrid controller for attitude and trajectory control. The systematic parametric analysis provides results of relevance to a number of near-Earth satellites and suggests a guideline for their semipassive control resulting in an extended lifespan.

Equations of Motion and Equilibria

For a rigid, nonspinning, axisymmetric satellite, librating in the gravity gradient field of the free molecular environment, the governing equations of motion in pitch, roll and yaw can be written as[6] (Fig. 1)

$$\psi''(1+eC_\theta)-2eS_\theta(\psi'+1)-2\phi'(\psi'+1)(1+eC_\theta)T_\phi+ 3K_i S_\psi C_\psi + B_{fp}A(1+eC_\theta)C_\psi(|C_\psi|+C_1 S_\psi)/C_\phi^2 = 0 \quad (1a)$$

$$\phi''(1+eC_\theta)-2eS_\theta\phi'+[(1+\psi')^2(1+eC_\theta)+ 3K_i C_\psi^2]S_\phi C_\phi = 0 \quad (1b)$$

$$\lambda'-(1+\psi')S_\phi = 0 \quad (1c)$$

where

$$B_{fp} = \rho_p C_D \varepsilon D_0 L_0 V_p^2/2I\dot{\theta}_p^2$$
$$A = [\{(1+e)/(1+eC_\theta)-R/r_p\}/(1-R/r_p)]^n * (1+2eC_\theta+e^2)(1+e)^2/(1+eC_\theta)^4 \quad (2)$$
$$C_1 = \pi D_0/4L_0 = \pi\{(1+K_i)/12(1+K_i)\}^{1/2}$$

Here n varies between -5 to -7 depending on the altitude.[14] Note, the yaw coordinate λ being cyclic the associated conjugate momentum is a constant of the motion leading to the first integral (1c).

As discussed by the authors,[6] a judicious arrangement of rotatable flaps attached to the satellite can provide, through aerodynamic reaction, a control moment in the individual degree of freedom thus giving a mechanism for damping the librational motion. In the present analysis, the controller characteristic is taken in the form

$$\tau_\psi = \mu_\psi \psi' + \nu_\psi(\psi-\gamma_\psi)$$
$$\tau_\phi = \mu_\phi \phi' + \nu_\phi(\phi-\gamma_\phi) \quad (3)$$

where μ_i, ν_i are positive proportionality constants representing gains of the system and γ_i, the position control parameters, impart versatility in attaining any arbitrary spatial orientation.

As can be expected, physical characteristics of the controller impose limits on the maximum available corrective moments

$$|\tau_\psi| \leq \tau_{\psi,\max}; \quad |\tau_\phi| \leq \tau_{\phi,\max} \quad (4)$$

and

$$\tau_{i,\max} = A\tau_{p,i} \quad i = \psi, \phi$$

$\tau_{p,i}$ denoting the maximum torque component at perigee depends on the flap arrangement and areas, satellite inertia, perigee height, and lift and drag characteristics. For example, a satellite with $I = 600$ slug ft^2, at $r_p = 200$ miles ($\rho = 3 \times 10^{-13}$ slug/ft^3) having one 3 ft × 3 ft flap with moment arm of 5 ft generating torque due to the lift only ($C_{L\max} = 0.2$)[14] has $\tau_{p,i} = 1$.

The second-order, coupled, nonlinear, nonautonomous equations of motion (1) are not amenable to any simple analytical approach. A numerical technique, therefore, is resorted to. Before solving the equations, however, some insight into the system behavior can be achieved by analyzing the equilibrium positions.

At equilibrium, the system potential is an extremum. Hence from Eqs. (1)

$$-2eS_\theta + 3K_i S_{\psi e} C_{\psi e} + B_{fp}A(1+eC_\theta) * C_{\psi e}(|C_{\psi e}|+C_1 S_{\psi e})/C_{\phi e}^2 + \nu_\psi(\psi_e-\gamma_\psi) = 0 \quad (5a)$$

$$[(1+eC_\theta)+3K_i C_{\psi e}^2]S_{\phi e}C_{\phi e} + \nu_\phi(\phi_e-\gamma_\phi) = 0 \quad (5b)$$

Thus the satellite attains the equilibrium position (ψ_e, ϕ_e), which is a function of all the system parameters except μ_i. It is apparent that in elliptical orbits the equilibrium configuration becomes a function of θ, hence a limit cycle motion in steady state is anticipated.

Nonuniqueness of the solution of the transcendental equation (5) leads to difficulties even for $e = 0$. Fortunately, application of the Routh criterion[15] shows that only one solution is stable.

Linearizing the equations of motion (1) about an equilibrium position, the corresponding characteristic equation can be written as

$$A_1\lambda^4 + A_2\lambda^3 + A_3\lambda^2 + A_4\lambda + A_5 = 0 \quad (6)$$

where

$$e = \psi_e' = \phi_e' = 0$$
$$A_1 = 1$$
$$A_2 = \mu_\psi + \mu_\phi$$
$$A_3 = 3K_i C_{\psi e}^2 - B_f(S_{\psi e}^2 - C_1 C_{\psi e}^2)/C_{\phi e}^2 + \nu_\psi + \nu_\phi + C_{\phi e}^2(1+3K_i C_{\psi e}^2) + \mu_\psi\mu_\phi + 4S_{\phi e}^2$$
$$A_4 = \mu_\psi\{C_{\phi e}^2(1+3K_i C_{\psi e}^2)+\nu_\phi\} + \mu_\phi\nu_\psi - 6K_i S_{\psi e}^2 S_{\phi e}^2$$
$$A_5 = \{3K_i C_{\psi e}^2 - B_f(S_{\psi e}^2 - C_1 C_{\psi e}^2)/C_{\phi e}^2 + \nu_\psi\} \times \{C_{\phi e}^2(1+3K_i C_{\psi e}^2)+\nu_\phi\}$$

For stable (ψ_e, ϕ_e)[15]

$$A_i \geqq 0 \quad i = 1, 2, 3, 4, 5$$
$$(A_2A_3 - A_1A_4)A_4 - A_2^2A_5 \geqq 0 \quad (7)$$

It may be pointed out that in the absence of an aerodynamic controller, the stable position is given by[5]

$$\phi_e = 0, \quad \psi_e = \tan^{-1}\{-B_f/(3K_i + B_f C_1)\} \quad (8)$$

The satellite deviates from the local vertical, at a decreasing rate, with increasing B_f, especially for smaller K_i (Fig. 2a). At $B_f = 4$, most satellites attain $\psi_e \simeq -50°$. Also, shorter satellites are relatively less affected by the atmosphere.

Introduction of a controller with suitable parameters can correct errors of large magnitudes and yield any desired stable configuration as seen in Fig. 2(b–e). The plots are obtained by setting ψ_e and ϕ_e equal to the position demanded. This leads to a relationship between the system parameters in Eq. (5) which also must satisfy the stability conditions (7). For example, to attain stable local vertical configuration, the controller constants should be such that (Fig. 2b)

$$\nu_\psi\gamma_\psi = B_f$$
$$\nu_\phi\gamma_\phi = 0 \quad (9)$$

Similarly, the local horizontal ($\psi_e = \pi/2$) orientation corresponds to $\gamma_\psi = \pi/2$ while $\phi_e = \pi/2$ is a singularity of the system. The parameters depend on the satellite inertia for any intermediate configuration. Figures 2c and 2d show combinations of γ_ψ and ν_ψ for various ψ_e keeping $\phi_e = 0$, and of γ_ϕ and ν_ϕ for various ϕ_e keeping $\psi_e = 0$, respectively. Negative values of ν (dotted lines) may not be useful because of: a) the nonunique character

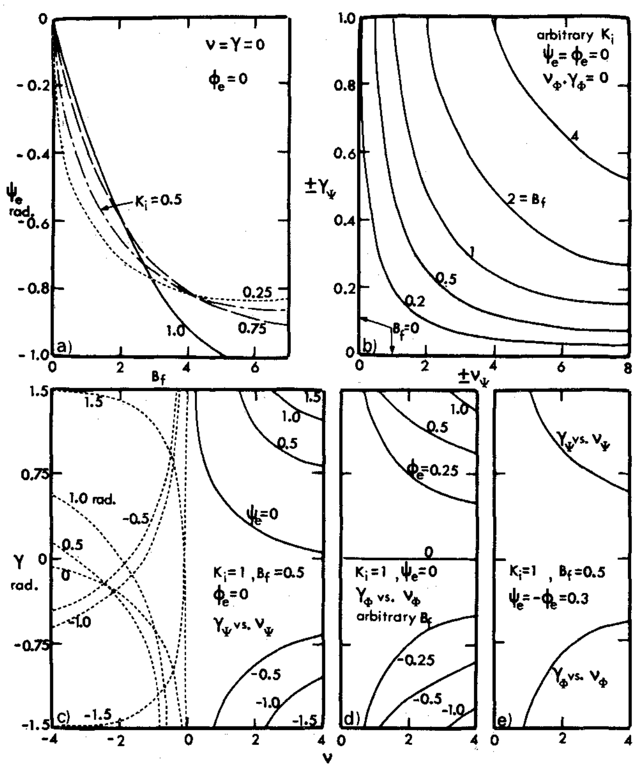

Fig. 2 Stable equilibrium positions in circular orbit: a) effect of satellite parameters in absence of control; b) conditions for local vertical configuration; c) planar inclinations; d) transverse inclinations; e) arbitrary orientation.

for a given combination of v and γ; b) their sensitivity to changes in v; c) possibilities of instability.

It is observed that with increasing v, γ decreases in magnitude. Larger values are required for larger B_f, ψ_e or ϕ_e. The sign of γ is the same as that of the corresponding equilibrium position. Plots similar to those in Fig. 2e can be generated for any desired position.

Optimization

The need for controller optimization can not be over emphasized, as it represents efficiency, weight saving and the best use of the available power—an expensive commodity aboard a spacecraft. The problem, however, is complicated by the presence of nonlinearities, coupling and a large number of parameters. The following observations may make the analysis manageable.

a) As the librational frequency and amplitude in the two degrees of freedom are of the same order of magnitude,[5,6,16] and the coupling keeps the cross motions sustained, μ_ψ and v_ψ may be chosen equal to μ_ϕ and v_ϕ, respectively.

b) The optimization need be carried out only for μ_ψ and v_ψ [γ_i is found from Eq. (5)] because satellites are more sensitive to planar disturbances[5] and exhibit planar limit cycle behavior in elliptical orbits.[6]

c) Since a librational motion in a circular orbit can be damped completely, T, the transient time to return to the equiblibrium configuration, may be used as the optimization criterion. For elliptical trajectories, however, a limit-cycle amplitude (or the maximum deviation, Δ, from a desired orientation during steady state) may form a suitable basis to the same end.

Figure 3 indicates the optimization procedure. First a range of parameters is selected and the stable region corresponding to a desired orientation is established using the Routh criterion [Eq. (7)]. Next, for systematically varied values of μ and v, the equations of motion (1) with an initial disturbance are solved. The damping time T and the maximum steady-state deviation Δ are noted for circular and elliptical orbits, respectively. The curves in the figure represent loci of specified T and Δ. The steepest paths (dotted lines) correspond to optimum combinations of the parameters.

Integration of the equations of motion was accomplished numerically using the Adams-Bashforth predictor-corrector method with the Runge-Kutta starter.[17] A step-size of 3° gives sufficiently accurate results for small values of the controller parameters. For larger gains, however, as in the case of elliptical orbits, a smaller step-size of 1° was necessary.

It may be noted that the values of γ_i change with v_i for a given equilibrium configuration. In elliptical orbits, difficulties are primarily due to the nonautonomous character of the equations by their dependence on θ. The above optimization, therefore, is carried out by keeping the γ_i constant and equal to their values at $\theta = 0$. The γ optimization will be discussed in the following section.

Discussion of Results

Optimization of the Controller Performance

The gravity-gradient torque tends to align the minimum moment of inertia axis along the local vertical ($\psi = \phi = 0$). With an aerodynamic controller for this purpose (Fig. 3a) the per-

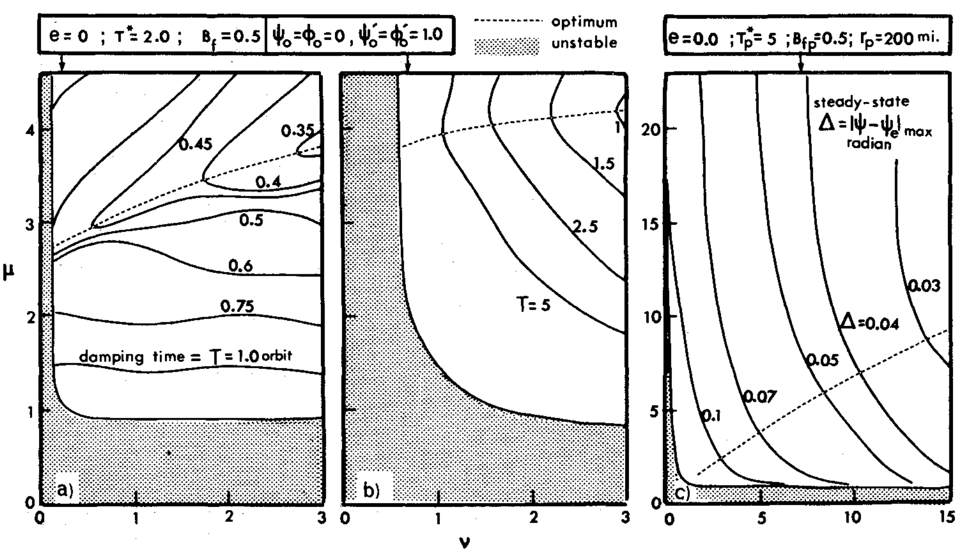

Fig. 3 Optimization of controller parameters ($\mu = \mu_\psi = \mu_\phi$ vs $v = v_\psi = v_\phi$ from Eq. (6); $K_i = 1.0$): a) $e = 0$, local vertical; b) $e = 0$, $\psi_e = \phi_e = 45°$; c) $e = 0.05$, local vertical.

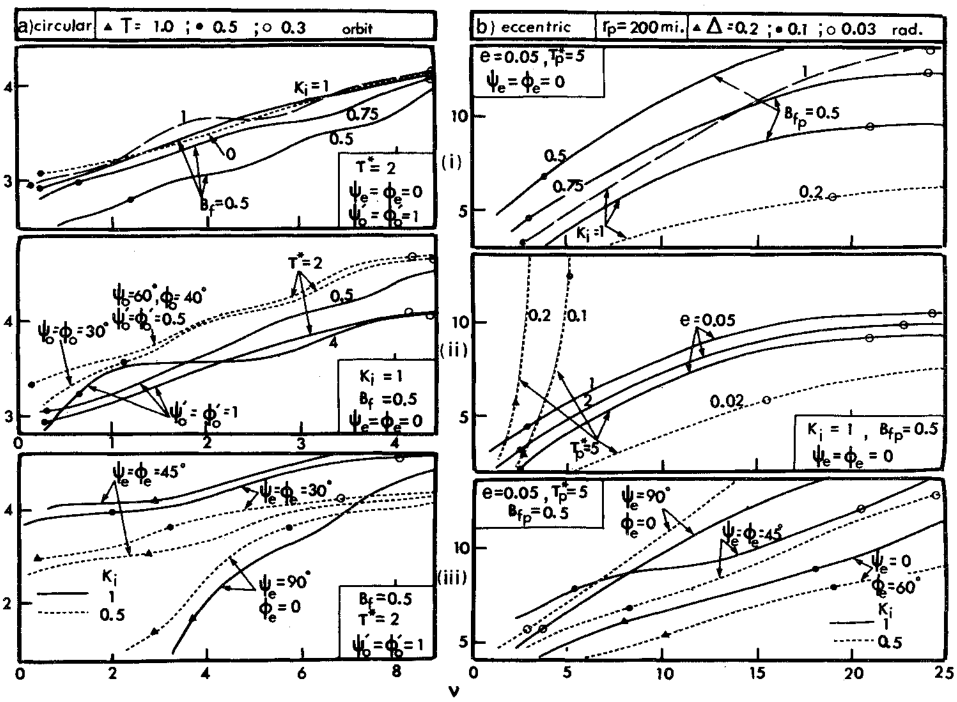

Fig. 4 Variation of optimum controller gains with system parameters in a) circular orbit and b) elliptical orbit showing the effect of (i) inertia and aerodynamic coefficients, (ii) disturbance, eccentricity and maximum torque, and (iii) equilibrium position.

formance in a circular orbit is considerably improved, i.e., the damping time is small even for small values of the controller parameter. For other positions (e.g., Fig. 3b), however, larger correcting torques are needed. The situation further deteriorates with orbital eccentricity (Fig. 3c) which represents a forcing function.

It may be pointed out that the variation of T and Δ along the steepest path is, in general, small compared to the variation in μ and ν thus emphasizing the difficulty in applying one of the steepest descent methods for optimization. On the other hand, it is encouraging to see that even for values of the parameters far from the optimum, the system performance remains acceptable thus permitting, during the control system design, compromises dictated by other considerations.

To establish the suitability of an aerodynamic controller over a wide range of system variables, a large number of plots similar to those in Fig. 3 were obtained. Figure 4 shows the corresponding optimum gains μ and ν.

In circular orbits (Fig. 4a) the damping time for satellites subjected to large disturbances ($\psi' = \phi' = 1.0$) can be as little as $\frac{1}{3}$ of an orbit in the μ-ν range indicated. In general, there seems to be only little variation with the aerodynamic coefficient B_f. A slender satellite (large K_i) performs better at the local vertical configuration [Fig. 4a(i)]. The steepest path is affected significantly by the initial conditions but the controller continues to exhibit good performance by damping very large disturbances. An improvement in the performance through increasing τ^*, the maximum available torque, is not significant beyond a certain value [e.g. $\tau^* = 4$ in Fig. 4a(ii)]. The last Fig. 4a(iii) shows the optimum variation of the controller's gains for several satellite orientations including the local horizontal—normally an unstable position. A strong dependence of the optimum paths on the equilibrium position is apparent. Although the torque requirement, particularly for slender satellites, is increased substantially, the ability to attain desired orientations imparts versatility to the satellites.

The effectiveness of the controller in elliptical orbits (Fig. 4b) appears to be promising. The pointing error can be reduced to a fraction of a degree within the range of μ, ν indicated. The plots show an increased sensitivity to the changes in parameters even in near circular orbits. Slender satellites (large K_i) with smaller aerodynamic coefficients B_f [Fig. 4b(i)] would be preferred for the local vertical orientation. The increased demands on the controller can be met by larger flaps (increasing τ_p^*). However, as seen in circular orbits, the performance improvement is not significant beyond $\tau_p^* = 5$ [Fig. 4b(ii)]. The atmosphere being effective only over a fraction of an elliptical orbit, the controllability reduces rapidly with increasing e. It may be noted that for higher eccentricities, larger values of ν affect the system adversely. A possibility of attaining any arbitrary configuration is exhibited by Fig. 4b(iii). The pointing errors for the local horizontal orientation are the smallest, perhaps due to the reduced effect of the forcing terms.

Any comprehensive study of this nature would involve a large number of plots. The figures here attempt to cover a sufficiently wide range of parameters to be useful during the preliminary design of a control system.

It should be noted that the pointing error Δ decreases at a diminishing rate with an increase in μ and ν. Beyond the range

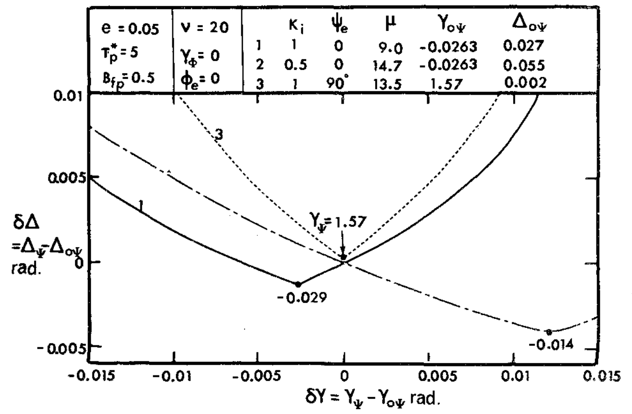

Fig. 5 Variation of the pointing error with the position control parameter.

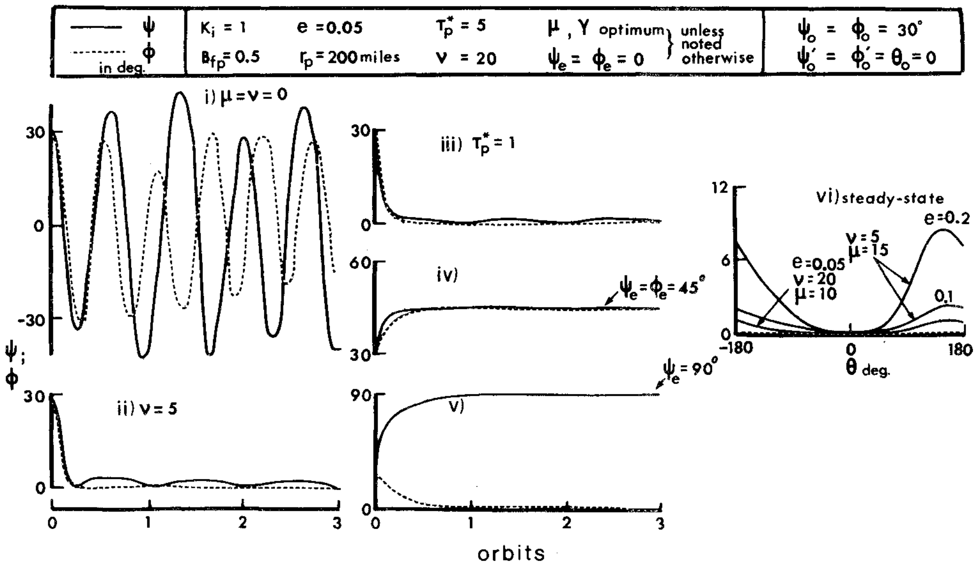

Fig. 6 Typical system responses.

indicated, it remains at about 0.015 in most cases. Further reduction in the pointing error can be achieved by optimizing γ which, in the present analysis, was arbitrarily chosen to correspond to the perigee condition. The procedure is indicated for three representative cases in Fig. 5 where the change in error ($\delta\Delta$) is plotted as a function of $\delta\gamma$ for a given set of system parameters. The lowest points represent the best values. It appears that in some cases (e.g., curve 2) an improvement in the pointing accuracy may be possible, however, in general, the choice of $\gamma = \text{constant} = \gamma_p$ in Eq. (5) represents a good approximation.

Librational Damping and Attitude Control

Having established a guideline for the design of the controller, the satellite motion itself may be of interest. The response of a satellite subjected to an arbitrarily large initial condition ($\psi_0 = \phi_0 = 30°$) is shown in Fig. 6. In the absence of a controller even a slender satellite executes librations of large magnitudes [Fig. 6(i)]. These motions are damped completely by a set of moderately-sized flaps in about $\frac{1}{3}$ of a circular orbit. However in elliptical orbits, only the cross plane motion returns to the equilibrium with the ψ motion attaining a steady-state limit cycle. Its amplitude is significant for smaller values of the controller parameters, namely μ, ν [Fig. 6(ii)] and τ_p* [Fig. 6(iii)] and for higher eccentricity, larger aerodynamic coefficient (B_{fp}) and smaller K_i (not shown). A proper choice of the controller characteristics can bring the error down to an acceptable level. It is of interest to recognize that the time required to achieve any arbitrary position continues to remain small [Fig. 6(iv)]. Although the limit cycle behavior persists for the intermediate position, especially with shorter satellites, it virtually disappeared as the satellite approached the local horizontal configuration [Fig. 6(v)].

It was inferred earlier that the controller may not be useful for higher eccentricity orbits which may be employed for increasing the satellite life time. The steady-state response over a range of eccentricity is compared, during one librational period (same as the orbital period) in Fig. 6(vi). Although, large deviations occur near apogee, the pointing errors in the vicinity of perigee ($|\theta| \leq 60°$) remain less than 1°, even for $e = 0.2$. This is encouraging because, in most cases, the desired observations by a satellite would be conducted near perigee for better resolution. For weather satellites, which take pictures at regular intervals and have large observational cone angles ($\approx 60°$), the pointing errors, even near apogee, may be acceptable. Furthermore, missions requiring a periodic sweep scanning of a given region can utilize the steady-state motion to advantage.

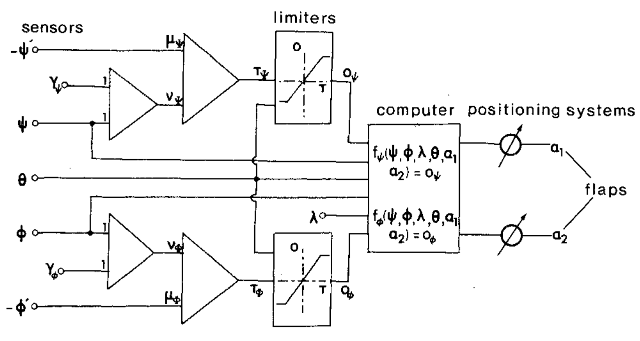

Fig. 7 Schematic diagram of the controller servosystem.

Controller Servo-System and Power Consumption

Figure 7 presents a schematic diagram of the controller servo-system. The state-variables are sensed, amplified and suitably added to yield τ_ψ and τ_ϕ. The limiters restrict their values to the corresponding maximums which vary with θ [Eqs. (4) and (5)]. The required torques are equated to the functions f_ψ and f_ϕ which depend upon flap arrangements, size, orbital position, librational angles and flap rotatons (α_i). The solution of these identities will be normally accomplished by an onboard analog or digital computer. The outputs drive the positioning systems resulting in appropriate flap rotation.

The design of a controller would be incomplete without consideration of the power required. The control system has to overcome the inertia forces, bearing friction,[18] and aerodynamic drag.[19] However, their proprtionality to the flaps' angular velocities, which are usually quite low, makes the power demand small. For instance, the peak power for a satellite with three 5 ft × 5 ft × $\frac{1}{8}$ in. aluminum plates, rotating in ball bearings, at 200 miles altitude is found to be less than 0.1 w. The efficiences of the positioning system and the control circuit also being high the system remains essentially a passive one.

The above analysis suggests that an effective, economical, quasi-passive, aerodynamic controller can be designed for most near-Earth ($r_p = < 350$ miles) multimission satellites.

Conclusions

The essential features of the analysis and important findings may be summarized as follows.

1) A quasi-passive, velocity and position sensitive aerodynamic

attitude controller for near-Earth satellites is developed. The controller enables the satellite to achieve any arbitrary orientation.

2) The stability of the static equilibrium orientations is established using the Routh criterion. In elliptical orbits, the equilibrium configuration is a function of the orbital position leading to a steady-state limit-cycle motion.

3) For optimization, the number of controller parameters is reduced through some logical assumptions. An optimum relationship between $\mu(=\mu_{\psi}=\mu_{\phi})$ and $v(=v_{\psi}=v_{\phi})$ is established on the basis of the transient-damping time and the steady-state pointing errors in circular and elliptical orbits, respectively. A systematic variation of the satellite orbit parameters (i.e., K_i, e, r_p, B_f, ψ_e, ϕ_e, τ) yields an operational region in the μ-v plane.

4) A response analysis shows the damping time to be as little as $\frac{1}{3}$ orbit and pointing error to be a fraction of a degree. For high eccentricity orbits the deviation continues to remain small around the perigee

5) Having established the effectiveness of the concept, a schematic diagram of the controller circuit is presented. The transcendental character of the moment equations may present a problem in controller mechanization.

Finally, it may be mentioned that the effectiveness of the controller can be further improved by combining it with a solar-controller.[4,8] A set of flaps can be suitably modified to utilize solar pressure and aerodynamic forces at high and low altitudes, respectively. The same flaps may also be used to make these environmental forces compensate for any orbital decay, thus enhancing the satellite lifetime. It may also be possible to slowly change the orbital elements thus adding to the satellite's versatility in undertaking diverse missions. These exciting possibilities are under investigation.

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